

# On Anomaly Mediated SUSY Breaking

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## Abstract

A discrepancy between the Anomaly Mediated Supersymmetry Breaking (AMSB) gaugino mass calculated from the work of Kaplunovsky and Louis (hep-th/9402005) (KL) and other calculations in the literature is explained, and it is argued that the KL expression is the correct one relevant to the Wilsonian action. Furthermore it is argued that the AMSB contribution to the squark and slepton masses should be replaced by the contribution pointed out by Dine and Seiberg (DS) which has nothing to do with Weyl anomalies. This is not in general equivalent to the AMSB expression, and it is shown that there are models in which the usual AMSB expression would vanish but the DS one is non-zero. In fact the latter has aspects of both AMSB and gauge mediated SUSY breaking. In particular like the latter, it gives positive squared masses for sleptons.

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## I. INTRODUCTION

Supersymmetry breaking is generally thought of as taking place in a hidden sector and is communicated to the visible sector through some messenger fields. The latter may be the moduli of string theory which interact only with gravitational strength with the visible fields, or some other messenger sector that couples to the gauge fields and also to the supersymmetry breaking sector. The former mechanism may be called moduli mediated supersymmetry breaking (MMSB) also known as gravity mediated supersymmetry breaking (for a review see [1]). The latter is called gauge mediated supersymmetry breaking (GMSB - see [2] for a review). The advantage of the latter over the former mechanism is that generically MMSB has flavor changing (charge) neutral interactions and mass terms which need to be suppressed by some fine tuning at the  $10^{-3}$  level in order to agree with experiment, while GMSB is naturally flavor neutral since the gauge interactions are flavor blind.

An alternative to GMSB which shares its feature of being flavor blind but like MMSB originates in the supergravity sector was proposed in [3] [4] [5]. This mechanism has been called anomaly mediated supersymmetry breaking (AMSB). This depends on the so-called Weyl (or conformal) anomaly of supergravity (SUGRA) and appeared to depend on a particular formulation of supergravity - namely the so-called Weyl (or conformal) compensator formalism. This feature is rather puzzling and is clearly in need of some explanation. In fact in [6] there is an argument based on the standard formulation of SUGRA, for the AMSB gaugino masses, but not for the squark and slepton masses. In [7] on the other hand arguments are given for contributions to both gaugino and scalar masses based on the need to preserve supersymmetry, independently of any particular formulation of supergravity.

In this paper we will first argue that AMSB (for gaugino masses) is in fact implicit in an old paper of Kaplunovsky and Louis [8][28]. Two versions of the calculation were given there; one in the Weyl compensator formalism and the other in the standard SUGRA formalism. We show how the correct expression for the AMSB contribution to the gaugino masses emerges from the compensator formalism. Then we rederive this expression in the usual formulation (with the Weyl compensator set to unity). In this case the contribution comes from Jacobians in the measure coming from field redefinitions necessary to get to the Kaehler-Einstein frame. The point of this is that the F-term of the Weyl compensator is determined to have a value which is different from that given in [3][4] and [6][29] and we

discuss the reason for this difference and argue that the correct contribution to the Wilsonian action is given (implicitly) by [8]. Next we discuss the contribution to the gaugino masses pointed out by Dine and Seiberg [7](DS). We argue that while it is certainly present, it is a new effect and is not equivalent to the AMSB contribution. Finally we consider the AMSB argument for soft masses. We point out that this actually violates the Weyl invariance of this formulation of supergravity. Then we consider the argument given in [7] (DS). We generalize it to show how a contribution to both Higgs sector and squark and slepton sector masses can arise from this mechanism by following standard supergravity calculations. We claim though that the DS mechanism is a new one, i.e. is not equivalent to the AMSB argument, and in fact (in the presence of a “mu” term in the superpotential) gives an additional term. We also point out that it is possible to find models (see section IV) in which the usual AMSB expression vanishes but the DS effect does not. Furthermore we argue that when the DS effect is calculated by taking into account the fact that the wave function renormalization at scales below the Higgs expectation value depends on threshold effects, it is very similar to GMSB, and there is no problem with the slepton masses. Of course unlike in GMSB the gravitino mass is heavy, and sequestering [3] is still necessary in order to ensure that the classical contribution to the soft masses does not dominate the quantum effects. Finally in an appendix we discuss in a simplified (non-supersymmetric) context some issues relevant to Weyl transformations.

## II. WEYL ANOMALIES

The manifestly Weyl invariant formalism of  $N = 1$  supergravity (SUGRA) is given by the following action (with  $\kappa = M_P^{-1} = 1$ ,  $d^8z = d^4x d^4\theta$ ,  $d^6z = d^4x d^2\theta$ ).

$$S = -3 \int d^8z \mathbf{E} C \bar{C} \exp\left[-\frac{1}{3}K(\Phi, \bar{\Phi}; Q, \bar{Q}e^{2V})\right] + \left(\int d^6z 2\mathcal{E}[C^3 W(\Phi, Q) + \frac{1}{4}f_a(\Phi)\mathcal{W}^a\mathcal{W}^a] + h.c.\right) \quad (1)$$

$$= -3 \int d^6z 2\mathcal{E}\left(-\frac{\bar{\nabla}^2}{4} + 2R\right) C \bar{C} \exp\left[-\frac{1}{3}K(\Phi, \bar{\Phi}; Q, \bar{Q}e^{2V})\right] + \left(\int d^6z 2\mathcal{E}[C^3 W(\Phi, Q) + \frac{1}{4}f_a(\Phi)\mathcal{W}^a\mathcal{W}^a] + h.c.\right) \quad (2)$$

In the above action  $\Phi$ ,  $Q$  are respectively a set of chiral superfields representing the moduli and the MSSM matter fields,  $V$  is the gauge prepotential and  $\mathcal{W}_\alpha = (-\frac{\bar{\nabla}^2}{4} + 2R)e^{-2V}\nabla_\alpha e^{2V}$

is the associated gauge field strength.  $R$  is the chiral curvature superfield,  $\mathbf{E}$  is the full superspace measure and  $\mathcal{E}$  is the chiral superspace measure. The so-called torsion constraints of SUGRA are invariant under Weyl transformations (with a chiral superfield transformation parameter  $\tau$ ) some of which are explicitly given below.

$$\begin{aligned}\mathbf{E} &\rightarrow e^{2(\tau+\bar{\tau})}\mathbf{E}, & \mathcal{E} &\rightarrow e^{6\tau}\mathcal{E}, \\ \nabla_\alpha &\rightarrow e^{(\tau-2\bar{\tau})}(\nabla_\alpha - \dots), & V &\rightarrow V, \\ \Phi &\rightarrow \Phi, Q \rightarrow Q, & \mathcal{W}_\alpha &\rightarrow e^{-3\tau}\mathcal{W}_\alpha.\end{aligned}\tag{3}$$

The Weyl compensator  $C$  with the transformation rule

$$C \rightarrow e^{-2\tau}C,\tag{4}$$

is then introduced in order to have a manifestly Weyl invariant action. Note that since  $C$  comes into the Kaehler potential in the form  $\ln C + \ln \bar{C}$  it is not a propagating field and the theory is completely equivalent to the usual formulation of supergravity. However this will remain true for the quantum theory (meaning the Wilsonian effective action rather than the 1PI effective action) only to the extent that this Weyl invariance can be preserved. Any violation of this invariance will result in the propagation of this degree of freedom and hence produce a theory which is inequivalent to the original supergravity. It should be stressed that provided supergravity is not explicitly broken, the above formalism gives the most general action, at the two derivative level, for a local supersymmetric theory coupling pure supergravity to a locally gauge invariant theory of chiral scalar superfields and gauge superfields.

The Weyl symmetry is anomalous at the quantum level because the path integral measure is not invariant under these transformations. The transformation of the measure can be obtained from the associated chiral anomaly [9][8][10] and takes the form

$$[d\Psi] \rightarrow [d\Psi] \exp \left\{ \frac{3c_a}{16\pi^2} \int d^6z 2\mathcal{E}\tau\mathcal{W}^a W + h.c. \right\}.\tag{5}$$

Here the anomaly coefficient is given by

$$c_a = T(G_a) - \sum_r T_a(r)\tag{6}$$

and  $T(G_a), T_a(r)$  are the trace of a squared generator in the adjoint and the matter representation  $r$  of the gauge group  $G_a$ . For future use we also give here the 1-loop  $\beta$ -function

coefficient

$$b_a = 3T(G_a) - \sum_r T_a(r). \quad (7)$$

This anomaly needs to be cancelled since the theory needs to retain this local Weyl invariance and  $C$  is a spurious degree of freedom. This is done by the replacement [8]

$$f_a(\Phi) \rightarrow \tilde{f}(\Phi, C) \equiv f_a(\Phi) - \frac{3c_a}{8\pi^2} \ln C, \quad (8)$$

and it is easily seen from the transformation rules for  $\Phi, C$ , that the anomaly is cancelled. This is essential (as stressed in [8]) in order to have equivalence between the usual (manifestly supersymmetric) formulation of SUGRA where the Weyl compensator is gauge fixed to  $C = 1$  and the Einstein-Kaehler frame action (with Einstein gravity, canonical gravitino kinetic and canonical Kaehler matter kinetic terms). The latter corresponds to the gauge fixing

$$\ln C + \ln \bar{C} = \frac{1}{3} K|_{Harm} \quad (9)$$

The instruction on the left hand side means that the chiral plus anti-chiral pieces are to be taken (i.e. in components the lowest component the  $\nabla_\alpha, -\frac{1}{4}\nabla^2$  and their hermitian conjugates are to be retained [30]). This is essentially the same as going to the Wess-Zumino gauge for the hermitian superfield  $K$ .

Let us now expand the Kaehler potential in terms of the matter fields by writing

$$K(\Phi, \bar{\Phi}; Q, \bar{Q}e^{2V}) = K_m(\Phi, \bar{\Phi}) + Z_{IJ}(\Phi, \bar{\Phi}) \bar{Q}^{\bar{J}} e^{2V} Q^I + \dots \quad (10)$$

The first term in the action (1) then becomes

$$\int d^8 z \mathbf{E} C \bar{C} [-3e^{-\frac{1}{3}K_m(\Phi, \bar{\Phi})} + e^{-\frac{1}{3}K_m(\Phi, \bar{\Phi})} Z_{IJ} \bar{Q}^{\bar{J}} e^{2V} Q^I + \dots]. \quad (11)$$

To get canonical normalization for the matter fields we need to do a field redefinition. For simplicity consider the case of one matter field multiplet in a representation  $r$ . The kinetic term is contained in

$$\int d^8 z \mathbf{E} C \bar{C} e^{-K_m} Z_r(\Phi, \bar{\Phi}) \bar{Q} e^{2V} Q, \quad Z_r^\dagger = Z_r.$$

Under the transformation  $Q \rightarrow e^{\tau_Z} Q$  (where  $\tau_Z$  is chiral) with all other fields fixed, the path integral measure acquires a factor

$$\exp \left\{ -\frac{T_a(r)}{16\pi^2} \left( \int d^6 z 2\mathcal{E} \tau_Z \mathcal{W}^a \mathcal{W}^a + h.c. \right) \right\} \quad (12)$$

This implies that under this transformation the gauge coupling function in the quantum action becomes

$$H_a(\Phi, C, \tau_Z) \equiv \tilde{f}_a(\Phi, C) - \frac{T_a(r)}{4\pi^2} \tau_Z = f_a(\Phi) - \frac{3c_a}{8\pi^2} \ln C - \frac{T_a(r)}{4\pi^2} \tau_Z \quad (13)$$

and the matter kinetic term becomes

$$\int d^8z \mathbf{E} \mathbf{C} \bar{\mathbf{C}} e^{-\frac{1}{3}K_m} Z e^{\tau_Z + \bar{\tau}_Z} \bar{Q} e^{2V} Q.$$

To get canonical normalization for the matter kinetic term we need to put

$$\tau_Z + \bar{\tau}_Z = \ln(C \bar{C} e^{-\frac{1}{3}K_m} Z_r) |_{\text{Harm}} \quad (14)$$

where the instruction on the right hand side means that the equality holds only for its harmonic part. Defining

$$h_a = H_a|, \quad h_{aR} = \Re H_a|,$$

the gauge coupling and the gaugino mass are given by (see for example [11] equation (G.2))

$\frac{1}{g_a^2} = h_{aR}$  and

$$m_a = h_{aR}^{-1} \Re(F^i \partial_i h_a + F^C \partial_C h_a + F^{\tau_Z} \partial_{\tau_Z} h_a). \quad (15)$$

Using (13) and (14) we then have

$$\frac{1}{g_a^2} = h_{aR} = (\Re f(\Phi) - \frac{b_a}{16\pi^2} \ln(C \bar{C}) - \frac{T_a(r)}{8\pi^2} \ln(e^{-\frac{1}{3}K_m} Z_r)) \quad (16)$$

$$= \Re f(\Phi)| - \frac{c_a}{16\pi^2} K_m| - \frac{T_a(r)}{8\pi^2} \ln Z_r| \quad (17)$$

The last expression is valid in the Einstein frame and we used (the lowest component of) (9) to obtain it. This is of course exactly the expression given in [8] (see equation C.15) evaluated at the cutoff scale and ignoring the term proportional to  $\ln \Re f_a$  which comes from rescaling to get the canonical kinetic term for the gauge potential. It should be stressed here that in [8] these expressions were also evaluated directly by explicit computations which showed that they are independent of whether or not a manifestly supersymmetric regulator was used, and confirmed the argument using the Weyl anomaly. Note also that the various scalar fields are to be evaluated at the minimum of the potential and in particular we have assumed that MSSM fields  $Q$  are set to zero at this point (so that  $K|_0 = K_m|_0$  for instance). The formula can be easily corrected if some of these fields are Higgses which have non

vanishing vacuum expectation values. Similarly the gaugino masses are given by

$$\frac{m_a}{g_a^2} = \Re[F^i \partial_i f_a(\Phi)] - \frac{b_a}{8\pi^2} \frac{F^C}{C} - \frac{T_a(r)}{4\pi^2} F^i \partial_i (\ln(e^{-\frac{1}{3}K_m} Z_r)) \quad (18)$$

$$= \Re[F^i \partial_i f_a(\Phi)] - \frac{c_a}{8\pi^2} F^i \partial_i K_m - \frac{T_a(r)}{4\pi^2} F^i \partial_i (\ln Z_r) \quad (19)$$

The sum over  $i$  it should be recalled goes over all the moduli (which are of course gauge neutral) and in the general case of more than one matter representation a sum over  $r$  is implied. Also to go from the first line to the second in the above expression we used the F-component of (9). The F-component of the moduli fields are as usual given by the formula

$$F^i = -e^{K/2} K^{i\bar{j}} D_{\bar{j}} \bar{W}. \quad (20)$$

At this point it behooves us to explain the differences between (19) and what has appeared before in the literature. In [3] and [4] it is asserted that  $F^C/C = m_{3/2}$ , whereas here (following [8]) it is fixed by the Einstein-Kaehler gauge condition (9). In [6] the formula that is given for the gaugino mass is (after adding the classical piece to equation (4) of that paper and changing the normalizations to agree with ours)

$$\frac{m_a}{g_a^2} = \Re[F^i \partial_i f_a(\Phi)] - \frac{1}{8\pi^2} (b_a m_{3/2} + c_a F^i \partial_i K_m + 2T_R F^i \partial_i \ln Z_r), \quad (21)$$

(though as the authors observed the calculation is sensitive to high scale effects). This formula could be obtained from our formula (18) if instead of using (the F-term of) equation (9) we used the formula

$$\frac{F^C}{C} = m_{3/2} + \frac{1}{3} F^i K_i. \quad (22)$$

In order to understand the meaning of one choice over the other it is instructive to first consider the equation of motion for the  $C$  field. Take the second form of the action (2) and vary it with respect to  $C$  to get

$$(-\frac{\bar{\nabla}^2}{4} + 2R)\bar{C} \exp[-\frac{1}{3}K(\Phi, \bar{\Phi}; Q, \bar{Q}e^{2V})] + C^2 W = 0 \quad (23)$$

Taking the lowest component of this equation and taking the value of  $C|$  from (9)) we get (ignoring fermionic terms)

$$\frac{\bar{F}^{\bar{C}}}{C} + 2R| = e^{K/2} W| + \frac{1}{3} \bar{F}^{\bar{i}} K_{\bar{i}} \quad (24)$$

So in the Einstein-Kaehler gauge, i.e. using the F-component of (9), this equation just determines the (lowest component) of the chiral curvature,  $2R| = e^{K/2} W| = m_{3/2}$ . The

equation (22) would be compatible with the equation of motion for  $C$  only in a gauge in which  $R| = 0$ .

The fact that the correct value of the AMSB contribution to the gaugino mass is given by (19) can also be seen in a different way - one that does not depend on the Weyl invariance argument of Kaplunovsky and Louis [8] and would be equivalent to the alternate argument given there [31]. In other words we will just use the standard supergravity formulation which corresponds to taking the gauge  $C = 1$  in (1). In this case to get to the Einstein-Kaehler gauge we need to make a Weyl transformation (3) with

$$2\tau + \bar{2}\tau = \frac{K_m}{3}|_{Harm}. \quad (25)$$

From (5) we see that this is tantamount to making the replacement

$$f_a(\Phi) \rightarrow f_a(\Phi, \tau) = f_a(\Phi) - \frac{3c_a}{4\pi^2}\tau.$$

The matter kinetic terms are now  $\int d^8z \mathbf{E} Z_r(\Phi, \bar{\Phi}) \bar{Q} e^{2V} Q$ . Next we need to do redefine the matter fields  $Q$  to get canonical normalization for them. This corresponds to the transformation  $Q \rightarrow e^{\tau_Z} Q$  with

$$\tau_Z + \bar{\tau}_Z = \ln(Z_r)|_{Harm}. \quad (26)$$

Again there is a contribution from the measure - namely (12), so that the effective gauge coupling function is finally

$$H_a(\Phi, \tau, \tau_Z) = f_a(\Phi) - \frac{3c_a}{4\pi^2}\tau - \frac{T_a(r)}{4\pi^2}\tau_Z. \quad (27)$$

Using (25)(26) and taking the F-component we again get (19).

It should be stressed that this contribution to the gauge coupling function has nothing to do with renormalization group running and the beta-function. The formulae (16)(19) are statements about the theory at the Wilsonian cutoff (say  $\Lambda$ ) where  $f_a$  is defined as the gauge coupling function in the original SUGRA frame. If we change the cutoff (say from  $\Lambda$  to  $\mu$  then to one-loop order we have

$$H_a(\Phi, \tau, \tau_Z, \mu) = H_a(\Phi, \tau, \tau_Z, \Lambda) - \frac{b_a}{8\pi^2} \ln \frac{\Lambda}{\mu} \quad (28)$$

It should be noted that while this last term contributes to the evolution of the gauge coupling, the ratio of the gaugino mass to the squared coupling is independent of the running since



the RG running term is a constant and only contributes to the lowest component of the superfield gauge coupling (however see [7] and the discussion below).

Of course as with all our previous considerations (18)(19)(27) are valid only for the Wilsonian coupling function which is not renormalized beyond one loop. This function however is not the physical coupling since the kinetic terms for the gauge fields is not canonically normalized. In order to get the physical coupling [32] we need to make a further transformation by a chiral superfield  $\tau_v$ ,

$$V = e^{(\tau_v + \bar{\tau}_v)/2} V_c, \quad (29)$$

such that the gauge field term in the action

$$\frac{1}{4} \int d^6 z \mathcal{E} \tilde{H}_a \mathcal{W}^a (e^{\tau_v + \bar{\tau}_v}/2 V_c) \mathcal{W}^a (e^{\tau_v + \bar{\tau}_v}/2 V_c) \quad (30)$$

is canonically normalized. Here we have redefined  $H_a$  to include a term coming from the measure so that  $\tau_v$  is to be determined from the equation

$$\Re \tilde{H}_a \equiv \Re H_a - \frac{T_a(G_a)}{8\pi^2} 2\Re \tau_v = e^{-2\Re \tau_v} \equiv \frac{1}{g_{phys}^2}, \quad (31)$$

so that the gauge field kinetic terms have canonical normalization. Combining the equations (25, 26, 27, 28, 31) gives us the NSVZ equation Novikov et al. [12] for the physical coupling in a locally supersymmetric theory.

Now let us comment on the calculation of [6] which is done in the  $C = 1$  gauge. This is based on the 1PI effective action of [13] where the non-local term

$$\begin{aligned} \Delta L = & -\frac{g^2}{(16\pi)^2} \int d^2\theta 2\mathcal{E} \mathcal{W} \mathcal{W} \frac{4}{\square} \left(-\frac{\bar{\nabla}^2}{4} + 2R\right) \\ & \{b_a 4\bar{R} + \frac{T_a(r)}{3} \nabla^2 K + T_a(r) \nabla^2 \ln Z_r\} + h.c. \end{aligned} \quad (32)$$

is added. Here  $\square$  is the flat-space Laplacian. This non-local action is designed to reproduce the super-Weyl anomalies that we have discussed and it is globally supersymmetric but is not locally supersymmetric (it is actually not generally covariant). Such a non-local action could have local ambiguities which need to be fixed by some criterion. The value of the gaugino mass coming from (32) is what is given in [6] and quoted in equation (21). In fact there is a simpler way of deriving this same result - with a similar problem. Thus instead of just adding the  $-\frac{3c_a}{8\pi^2} \ln C$  term as in [8] to cancel the anomaly, one again works in the

$C = 1$  gauge and adds a term

$$- \frac{3c_a}{8\pi^2} \ln \phi \quad (33)$$

where  $\phi = \mathcal{E}^{1/3}$ , to *reproduce* the Weyl anomaly [33]. Then in (18) the  $\ln C$  term would be replaced by a  $\ln \phi$  term. Then noting that (since  $-\nabla^2 \mathcal{E}/4 = 6e\bar{R}$ ) (see for example [11] equations (20,21,22)) we have

$$\frac{F^\phi}{\phi} = 2\bar{R} = m_{3/2} + \frac{1}{3} F^i K_i, \quad (34)$$

where the last equation is equation (24) in the  $C = 1$  Weyl gauge. In other words the effect of replacing  $C$  by  $\phi$  is to use (22) in (18) giving us (21) as we argued earlier. However  $\phi$  unlike  $C$  is not really a chiral scalar. Although it is chiral,  $\phi^3 = \mathcal{E}$  is a chiral density and so the term we added, like the non-local action of [13] but unlike the term  $\ln C$ , is globally but not locally supersymmetric. Also this term gives an unusual term proportional to  $\ln e$  in the expression for the coupling. This clearly shows that we have introduced a diffeomorphism anomaly though of course in flat space it is zero. Similarly the non-local addition (32) gives a non-local contribution to the gauge coupling.

We conclude that the correct anomaly mediated contribution to the gaugino mass in the Wilsonian action is given by (19). In fact as we showed in the discussion leading to (27) the calculation just depends on using the appropriate expressions for the relevant Jacobians in going to the Einstein-Kaehler frame with canonical normalization for the matter fields, and is completely unambiguous.

However an additional contribution to the gaugino mass arises from an effect first noticed by Dine and Seiberg [7](DS). This is usually ignored since the vacuum expectation values of the MSSM fields are set to zero. However some of these fields (Higgses) have non-zero expectation values in the physical vacuum and these authors propose that in effect the RG scale  $\mu^2$  should be replaced by  $\chi_+ \chi_-$  where  $\chi_\pm$  are a pair of Higgs fields (for instance they could be the MSSM charge neutral Higgs superfields  $h_{u,d}^0$  which have equal and opposite hypercharge). In fact this is what should be done in a background field calculation of the one-loop effective action. In this case the gauge coupling function  $H$  at the MSSM scale would have an additional term

$$H_a \sim \frac{b_a}{16\pi^2} \ln \frac{\chi_+ \chi_-}{\Lambda^2} \quad (35)$$

To preserve supersymmetry  $\chi_\pm$  must be the complete superfield. Of course as pointed out in [7] this formula is only valid in the Higgs phase of the theory. This then gives an additional

contribution to the gaugino mass

$$\frac{m_a}{g_a^2} \sim \frac{b_a}{16\pi^2} \Re\left(\frac{F^+}{\chi^+} + \frac{F^-}{\chi^-}\right) \quad (36)$$

We emphasize that this expression gives an unambiguous contribution to the gaugino mass since we are in the Higgs phase. Of course in the symmetric phase this expression would be of the form  $0/0$  and ambiguous, but in this phase eqn (35) would no longer be valid and one would need an explicit infra-red cutoff. In the MSSM for example this effect is present only in the physical Higgs vacuum.

To see what this contribution is in a concrete example note that after the various field redefinitions discussed earlier the MSSM fields have canonical normalization and in particular we may take (setting the Planck scale  $M_P = 1$ )

$$K \sim \chi_+ \bar{\chi}_+ + \chi_- \bar{\chi}_- + Q \bar{Q} \dots, \quad W \sim W_0 + m \chi_+ \chi_- + h \chi_+ Q^2. \quad (37)$$

The ellipses in  $K$  represent the hidden sector fields and  $W_0$  is the superpotential in the hidden sector with the hidden sector fields having a supersymmetry breaking minimum at some low scale generating a non-zero gravitino mass  $m_{3/2} = e^{K_0/2} W|_0$ . This is of course just a toy version of the MSSM with  $Q$  being the “top” quark/squark (with “hypercharge”  $-\frac{1}{2}$ ) whose loops can induce gauge symmetry breaking in the usual fashion (see for example [14], [15][16]). The actual situation in the MSSM is in fact a straightforward generalization of this. Thus after hidden sector supersymmetry breaking this model will be in the Higgs phase so that (35)(36) make sense and are unambiguous. As in the MSSM vacuum then  $\chi_{\pm} = v_{\pm} \neq 0$ , which may without loss of generality be chosen real (as in the MSSM) and  $Q = 0$ . Defining  $\frac{v_+}{v_-} = \tan \beta$  we get

$$\bar{F}^{\pm}|_0 = e^{K_0/2} (m v_{\mp} + v_{\pm} W|_0). \quad (38)$$

Defining  $\tilde{m} = e^{K_0/2} m$  we get from (36) the contribution

$$\frac{m_a}{g_a^2} \sim \frac{b_a}{8\pi^2} (m_{3/2} + \tilde{m} \operatorname{cosec} 2\beta). \quad (39)$$

If one ignored the “mu” term contribution (i.e. the second term in the paranthesis), it would seem that we have restored the  $O(m_{3/2})$  present in (21). However the origin of these terms is very different. Furthermore as we will see in the next section the interpretation of  $\chi^{\pm}$  as Higgs superfields will result in a further modification which will result in a formula

analogous to the one in GMSB[34]. Thus we conclude that the above effect is a new one which adds to the AMSB effect, which as argued earlier is actually given by (19) rather than (21). In fact as is evident from the above calculation it depends on the form of the visible sector superpotential. If there is no “mu” term as in the example considered by Dine and Seiberg [7] then the contribution is the same as that in the old AMSB calculations such as that of [6]. But if there is a “mu” term (as there must be in any realistic theory of low energy supersymmetry) then there is another term coming from the DS calculation that is not present in the old AMSB calculations.

### III. SOFT MASSES IN AMSB

In addition to a contribution to the gaugino mass, AMSB effects are supposed to contribute to the soft masses of MSSM scalar fields as well as to their couplings. Let us first review the usual argument. This may be motivated from the following observation for the gauge coupling superfield chiral scalar function  $H_a$ . Using the Weyl compensator formalism the Wilsonian coupling at some scale  $\Lambda$  can be written (by combining (13) and (III) as

$$H_a(\Phi, C, \tau_Z) = f_a(\Phi) - \frac{b_a}{8\pi^2} \ln C - \frac{T_a(r)}{4\pi^2} \ln(e^{-\frac{1}{3}K_m} Z_r) \quad (40)$$

where it is implied that only the lowest (whose phase is undetermined) and  $\theta$  and  $\theta^2$  components of the last term are taken. The gauge coupling function at some scale  $\mu$  is then given by adding the term  $-\frac{b_a}{8\pi^2} \ln \frac{\Lambda}{\mu}$  (as in (28)) giving the the coupling function at scale  $\mu$  as

$$H_a(\Phi, C, \tau_Z)_\mu = f_a(\Phi) - \frac{b_a}{8\pi^2} \ln(C\Lambda/\mu) - \frac{T_a(r)}{4\pi^2} \ln(e^{-\frac{1}{3}K_m} Z_r). \quad (41)$$

This might lead to the supposition that one should replace  $\Lambda/\mu$  by  $C\Lambda/\mu$  in the superfield functions that occur in the Wilsonian action evaluated at the scale  $\mu$ . In particular the wave function renormalization function  $Z(\Phi, \bar{\Phi}, \ln \frac{\Lambda}{\mu})$  at scale  $\mu$  might be replaced by  $Z(\Phi, \bar{\Phi}, \ln \frac{\Lambda|C|}{\mu})$ . If this is indeed justified then there would be an anomaly mediated contributions to the soft masses [3][5],

$$m^2 = -\ln Z|_{\theta^2\bar{\theta}^2} = -\frac{1}{4}|F^C|^2 \frac{d^2 \ln Z}{d \ln \Lambda^2}. \quad (42)$$

This would be the dominant contribution if the usual classical contribution (from the F-term of  $\Phi$  is suppressed by sequestering (see [3])). However the origin of the  $\ln C$  term and

the  $\ln \Lambda/\mu$  terms in (41) is completely different. The first exists even without any running i.e already at the high scale where the classical coupling is defined. It comes from the field redefinition/Weyl transformation Jacobians/anomalies. The second is a consequence of running. More importantly if one used the function  $Z(\Phi, \bar{\Phi}, \ln \frac{\Lambda|C|}{\mu})$  in the Wilsonian action then it is no longer invariant under the Weyl transformations and hence it would not be possible to remove  $C$  from the theory. In fact it is precisely the Weyl variation of the  $\ln(C\Lambda/\mu)$  term in (41) that guarantees the Weyl invariance of the quantum theory by canceling the Weyl anomaly. Finally the derivation of the gauge coupling function in the Einstein-Kaehler frame given in the discussion from (25) to (27), shows that the extra terms in the gauge coupling function are just a consequence of the field redefinitions. The apparent symmetry between the  $\ln C$  term and the RG term  $\ln \Lambda/\mu$  term has no physical significance. From the Wilsonian point of view the two scales  $\Lambda$  and  $\mu$  are both physical scales and should be measured in the same conformal gauge. Thus their ratio should be independent of the conformal gauge that is chosen. Indeed if one works in the  $C = 1$  gauge one can still derive the contribution to the gauge coupling function as we did above, but there is no field corresponding to  $C$  that can be used since the only other possibility, namely  $\phi$ , is not really a chiral scalar but a density, and as we pointed out earlier its use would violate local supersymmetry/general covariance.

The problem with the usual AMSB argument is that it is based on conformal invariance rather than Weyl invariance. Unlike the conformal invariance the Weyl invariance exists whether or not there are mass terms. This is because it involves transforming the metric whereas in the usual discussion Weyl invariance is spontaneously broken by restricting the argument to flat space. If this is done one loses sight of the (super) general covariance of the supergravity action. In other words the invariance in question involves transforming the background that is held fixed in the usual discussion. If the Weyl invariance of the action is violated (as would be the case if  $C$  dependence is introduced into the wave function renormalization then local supersymmetry will not be preserved.

An alternative mechanism for generating soft masses was given in [7] (DS). The mechanism is quite general but let us first discuss it within the context of the example given in that paper. The supergravity potentials are given by the following.

$$K = -3 \ln[1 - \frac{1}{3}K_v(\chi, \bar{\chi}) - \frac{1}{3}K_h(z, \bar{z})], \quad (43)$$

$$K_h = z\bar{z} - \frac{z^2\bar{z}^2}{\mu^2}, \quad (44)$$

$$K_v = Z(\chi\bar{\chi})\chi\bar{\chi}, \quad (45)$$

$$Z = 1 + \epsilon a_1 \ln(|\chi|^2/\Lambda^2) + \epsilon^2 a_2 \ln^2(|\chi|^2/\Lambda^2), \quad (46)$$

$$W = W_0 - M^2 z + W_v(\chi), \quad (47)$$

with  $M_P = 1$ ,  $M \ll \mu \ll 1$ ,  $\epsilon = g^2/16\pi^2$  and  $a_{1,2}$  are model dependent numbers. The constant  $W_0$  is tuned such that  $V_0 = 0$  and at the minimum we have (ignoring the matter sector)

$$F^z \simeq M^2, m_{3/2} = M^2/\sqrt{3}, z = z_0 \equiv \mu^2/2\sqrt{3} \ll 1. \quad (48)$$

The visible sector is assumed to be such that at the minimum of the combined potential  $F^\chi \ll M^2$  and  $\chi_0 \ll z_0$ . We also have near the minimum

$$\begin{aligned} K &\simeq K_v + K_h + \frac{1}{3}K_v K_h + \dots, K_{z\bar{z}} \simeq 1 + O(\mu^2), K_{\chi\bar{\chi}} \simeq 1 + O(\epsilon) \\ K_{v\chi} &\simeq \bar{\chi}, K_{h\bar{z}} \simeq z, K_{\chi\bar{z}} \simeq \frac{1}{3}\bar{\chi}z = -K^{\chi\bar{z}}. \end{aligned} \quad (49)$$

With  $\partial_i V_0 = V_0 = 0$ , the (squared) soft mass is essentially the Fermi-Bose splitting of the squared masses and is given by (see for example [11] p187-188)

$$\begin{aligned} \Delta m_{\chi\bar{\chi}}^2 &= M_{\chi\bar{\chi}}^2 - m_{\chi\bar{\chi}}^2 = e^{K_0} [-R_{\chi\bar{\chi}k\bar{l}} K^{k\bar{m}} K^{\bar{l}n} D_n W D_{\bar{m}} W + K_{\chi\bar{\chi}} |W|^2] \\ &= e^{K_0} [-(R_{\chi\bar{\chi}z\bar{z}} (K^{z\bar{z}})^2 |D_z W|^2 + R_{\chi\bar{\chi}\chi\bar{\chi}} (K^{\chi\bar{\chi}})^2 |D_\chi W|^2) \\ &\quad + K_{\chi\bar{\chi}} |W|^2 + O(\mu^2 m_{3/2}^2)]. \end{aligned} \quad (50)$$

In standard calculations of soft mass terms (see for example [17]) only the first term in the second line above is kept since SUSY breaking happens in the hidden sector and  $|D_\chi W|_0 = 0$ . However here the Kaehler metric is singular at  $\chi = 0$ , so there are extra terms if  $|D_\chi W|$  goes to zero no faster than linearly. We find

$$R_{\chi\bar{\chi}z\bar{z}} \simeq \frac{1}{3} K_{h\bar{z}\bar{z}} K_{v\chi\bar{\chi}}, R_{\chi\bar{\chi}\chi\bar{\chi}} \simeq K_{\chi\bar{\chi}} (2a_2 - a_1^2) \frac{\epsilon^2}{\chi\bar{\chi}}.$$

As expected (since  $K$  is of the sequestered form) the usual contribution vanishes. So we get (since  $K_{\chi\bar{\chi}} = 1 + O(\epsilon)$  and  $e^{K_0} \simeq 1$ ) for the normalized soft mass squared,

$$\begin{aligned} m_s^2 &\simeq -R_{\chi\bar{\chi}\chi\bar{\chi}} |F^\chi|^2 = -R_{\chi\bar{\chi}\chi\bar{\chi}} |K^{\chi\bar{\chi}}|^2 |D_\chi W|^2 \\ &\simeq (a_1^2 - 2a_2) \frac{\epsilon^2}{\chi\bar{\chi}} |\partial_\chi W_v + \bar{\chi} W|^2 = \epsilon^2 (a_1^2 - 2a_2) |m_{3/2} + O(\frac{\partial_\chi W_v}{\bar{\chi}})|^2, \end{aligned} \quad (51)$$

where in the last two steps we used (48) and (49). Note that all classical contributions to the soft masses cancel because of the sequestered form of the Kaehler potential . If there are no “mu” terms (i.e. terms of the form  $m\chi^2$ ) in  $W_v$  then we have the result of DS.

Let us compare this to the usual AMSB formula (42). If we assume that its F-term is given by (22)

$$\frac{F_C}{C} = m_{3/2} + \frac{1}{3}F^z\partial_z K = m_{3/2}(1 + O(\mu^2)). \quad (52)$$

Also

$$\gamma = -\partial \ln Z / \partial \ln \Lambda = 2\epsilon a_1 + 4\epsilon^2(2a_2 - a_1^2) \ln \frac{|\chi|}{\Lambda}, \quad (53)$$

where in the last step we used (46). This then gives

$$m_s^2 = \epsilon^2(a_1^2 - 2a_2)m_{3/2}^2 \quad (54)$$

in agreement with the DS calculation if there are no “mu” terms. However the appearance of a “mu” term contribution in the DS calculation means that it is not completely equivalent to AMSB. For instance if the “mu” term is fine-tuned to cancel exactly the  $\bar{\chi}W$  term (at the minimum) so that  $D_\chi W$  vanished quadratically with  $\chi$ , the DS contribution would be absent. Finally the AMSB argument for scalar masses would involve breaking the Weyl invariance while the DS calculation does not.

The above is valid for the Higgs fields of the low-energy theory but it is not clear from the above how the squarks and sleptons (which should have zero expectation values) should get DS type contribution to their mass. To see how this happens let us extend the DS toy model by adding a superfield  $Q$  (standing for a toy version of a quark or lepton superfield) which will have zero expectation value and no mass term but having a Yukawa interaction with the “Higgs” field  $\chi$ . Thus we replace the matter Kaehler potential by

$$K_v = Z(\chi\bar{\chi})(\chi\bar{\chi} + Q\bar{Q}) \quad (55)$$

where  $Z$  is again given by (46) and write the superpotential  $W_v$  in(47) as

$$W_v = \frac{m}{2}\chi^2 + h\chi Q^2 + \dots \quad (56)$$

with the ellipses containing terms which are higher order in the fields. Now we assume that the latter are such that the potential has a minimum (see also the discussion in the previous section) with

$$\chi_0 = \bar{\chi}_0 = v, \quad Q_0 = 0. \quad (57)$$

From the above we have  $F^{\bar{\chi}} = (\tilde{m} + m_{3/2})v$ ,  $R_{Q\bar{Q}\chi\bar{\chi}} = \epsilon^2(2a_2 - a_1^2)/v^2$ . Then following the same steps as in (50)(51) we get

$$\Delta m_{Q\bar{Q}}^2 = -R_{Q\bar{Q}\chi\bar{\chi}}|F^{\chi}|^2 = \epsilon^2(a_1^2 - 2a_2)(\tilde{m} + m_{3/2})^2. \quad (58)$$

Thus we do indeed have a contribution to the soft masses but again as was case with the DS contribution to the gaugino masses, it has nothing to do with Weyl anomalies. Furthermore unlike what is usually claimed as a contribution to the scalar mass from AMSB, the DS contribution fits naturally into the standard calculation of soft mass terms in supergravity.

#### IV. MODELS WITH $F^C = 0$ AND NON-ZERO DS EFFECT

As noted in [3], for the dominant contribution to the soft masses to be from AMSB the classical contribution from SUSY breaking in the hidden sector needs to be sequestered - as in equation (43) above. The same is obviously true for the alternative to AMSB, namely the DS version discussed above. A sequestered version that can naturally arise in type IIB string theory is one of the GKP-KKLT [18] [19] type with the visible sector being on a set of D3 branes. In such a model with just one Kaehler modulus  $T$  acquiring a non-zero F-term at the minimum (it is fine tuned by choosing fluxes and non-perturbative terms so that the cosmological constant is zero and SUSY breaking is only from this modulus) the soft masses will indeed be zero, and both the so-called  $A$  and  $B$  terms are also zero.

We first consider here the simplest version of this - namely the so-called no-scale model (which in type IIB is derived by GKP [18]). This illustrates the point, although of course the scale of SUSY breaking and the modulus  $T$  are not fixed. As is well known the soft masses and the  $A$  and the  $B$  terms, are all zero in such models (see for example [20] and references therein) even though supersymmetry is broken with a non-zero gravitino mass and a zero cosmological constant.

The point that we want to illustrate here is that when calculating the soft masses, the appropriate input from supergravity has to be taken for  $F^C$ . Thus consider the following toy model for the superpotential and Kaehler potential.

$$W = W_{mod} + mH^2 \quad (59)$$

$$K = -3\ln(T + \bar{T} - \frac{1}{3}H\bar{H}) \simeq K_{mod} + ZH\bar{H} + O(H^2\bar{H}^2) \quad (60)$$



with  $K_{mod} = -3\ln(T + \bar{T})$  and  $Z = 1/(T + \bar{T})$  and  $\partial_T W_{mod} = 0$ . The standard argument in supergravity consists of evaluating the usual expression for the potential for the chiral scalars and then extracting the scalar mass terms i.e. coefficients of the  $H\bar{H}$ ,  $HH$  terms, and one finds that they are zero.

What would the corresponding effective global calculation yield. In computing the potential one can of course ignore the chiral curvature ( $R$ ) terms and effectively work with the flat space Lagrangian

$$L = -3 \int d^4\theta C \bar{C} e^{-K/3} + (\int d^2\theta C^3 W + h.c.) \quad (61)$$

$$= -3 \int d^4\theta C \bar{C} e^{-K_{mod}/3} + (\int d^2\theta C^3 W_{mod} + h.c.) \quad (62)$$

$$+ \int d^4\theta \hat{H} \bar{\hat{H}} + (\int d^2\theta C m \hat{H}^2 + h.c.) \quad (63)$$

In the last two lines we have used the above toy model and rescaled (as is usual in AMSB type calculations)  $H \rightarrow \hat{H} \equiv CH$ . Now the usual discussion of AMSB proceeds from the last line. If this were whole story (as far the visible sector were concerned) there would be for instance a problem with the so-called  $B\mu$  term i.e. the coefficient of the  $H^2$  (where  $H$  refers to the scalar component) in the potential. For this would be then given by (see for example the review [21])  $F^C m$ . However the value of  $F^C$  needs to be fixed from the line (62) of this equation. In fact of course the first line (61) leads (upon elimination of  $C$ ) to the usual SUGRA potential and therefore to the result that all soft terms are zero.

Obviously one should get the same result from the second form (62) plus (63) of the Lagrangian. In this version the line (62) is used to get  $F^C$  (up to small corrections  $O(H^2)$ ) and this SUGRA input must be used to compute effects in the ‘MSSM’ sector of line (63). So from (62) we get as usual from the (lowest components of) the equations of motion for  $C$  and the chiral super fields,

$$\bar{F}^{\bar{C}} = C^2 e^{K/3} (W - \frac{1}{3} K_{\bar{j}} K^{\bar{j}i} D_i W) \quad (64)$$

$$= C^2 e^{K_{mod}/3} (W_{mod} - \frac{1}{3} \frac{3}{T + \bar{T}} \frac{(T + \bar{T})^2}{3} \frac{3W_{mod}}{(T + \bar{T})}) + O(H^2) = O(H^2), \quad (65)$$

since  $D_T W_{mod} = \partial W_{mod} - 3W_{mod}/(T + \bar{T}) = -3W_{mod}/(T + \bar{T})$  in this no-scale case. Thus the  $B\mu$  term is actually zero (to  $O(H^2)$ ) as are all other soft terms.

A similar situation exists for more realistic models where the  $T$  modulus is stabilized. The MSSM sector will have (schematically) quark/lepton superfields denoted by  $Q$  and Higgs

fields denoted by  $H$ . For the Kaehler potential we take

$$K = -3 \ln(T + \bar{T}) - \frac{1}{3}(H\bar{H} + Q\bar{Q}) - \ln(S + \bar{S}) + k(z, \bar{z}) \quad (66)$$

$$= K_{mod} + Z(H\bar{H} + Q\bar{Q}) + \dots \quad (67)$$

$$K_{mod} = -3 \ln(T + \bar{T}) - \ln(S + \bar{S}) + k(z, \bar{z}), \quad Z = \frac{1}{T + \bar{T}}. \quad (68)$$

For the moduli superpotential we take a GKP-KKLT [18, 19] form

$$W_{mod} = W_{flux}(S, z) + \sum_n A_n(S, z) e^{-a_n T}, \quad (69)$$

while for the 'MSSM' superpotential we take

$$W_{MSSM} = mH^2 + yHQQ$$

In the above  $S$  is the dilaton-axion superfield and  $z = \{z^r\}$  denotes the set of complex structure moduli and  $T$  is the Kaehler modulus of some Calabi-Yau orientifold (with  $h_{11} = 1$ ) compactification of type IIB string theory. Such a model can be realized as a generalization of those considered by GKP-KKLT [18][19]. Also the MSSM sector is located on a stack of D3 branes. The moduli potential is then

$$V_{mod} = \frac{e^{k(z, \bar{z})}}{(S + \bar{S})(T + \bar{T})^2} \left\{ \frac{1}{3} |\partial_T W_{mod}|^2 - 2\Re \partial_T W_{mod} \bar{W}_{mod} \right\} + |F^S|^2 K_{S\bar{S}} + F^z F^{\bar{z}} k_{z\bar{z}}. \quad (70)$$

Now one looks for a local minimum of this potential with zero cosmological constant (CC) and SUSY breaking only in the  $T$  direction, i.e.

$$V_{mod}|_0 = 0, \quad F|_0^S = F^z|_0 = 0, \quad F|_0 \neq 0. \quad (71)$$

There is certainly no obstruction to finding such a minimum and with a sufficient number of complex structure moduli and non-perturbative terms it is reasonable to expect that such a SUSY breaking minimum exists. The fine tuning condition for the CC now takes the form (at the above local minimum of the potential)

$$|D_T W_{mod}|_0^2 \frac{(T + \bar{T})_0^2}{3} = 3|W_{mod}|_0^2, \quad (72)$$

or taking the same phase as in the no-scale model we have

$$D_T W_{mod}|_0 (T + \bar{T})_0 = -3W_{mod}|_0. \quad (73)$$

It should be stressed that unlike in the case of the no-scale model (where these relations are automatic) in the present case they are fine tuned relations that are valid at the SUSY breaking local minimum (71). In effect the relation implies that we should fine tune such that  $\partial_T W|_0 = 0$  which is certainly possible if there are at least two non-perturbative terms in (69). Using (64) and (73) we get

$$F^C|_0 = C^2 e^{K_{mod}/3} (W_{mod} + D_T W|_{mod} \frac{T + \bar{T}}{3})|_0 = 0, \quad (74)$$

ignoring  $O(H^2)$  terms. Defining  $\hat{H} = CH$ ,  $\hat{Q} = CQ$  as in (63) we see that the effective ‘MSSM’ theory is given by (noting that  $e^{-K_{mod}/3} Z = (S + \bar{S})^{1/3} k^{-1/3}(z, \bar{z})$ )

$$L_{MSSM} = \int d^4\theta (S + \bar{S})^{1/3} k^{-1/3}(z, \bar{z}) (\hat{H} \bar{\hat{H}} + \hat{Q} \bar{\hat{Q}}) + \left\{ \int d^2\theta (m C \hat{H}^2 + y \hat{H} \hat{Q}^2) + h.c. \right\}. \quad (75)$$

Since this is independent of the SUSY breaking modulus  $T$ , and as we saw above  $F^C$  is also zero at the minimum of the moduli potential, all soft SUSY breaking terms are zero.

In addition to the vanishing of the classically generated soft terms, in this model the usual AMSB expression is also zero. The latter is obtained by inserting a wave function renormalization factor  $Z(\mu C/\Lambda)$  into the first term of (75) and this gives a contribution to the soft terms proportional to  $F^C$ . But since the latter is zero at the minimum of the moduli potential in this model, there is no such contribution. Nevertheless the DS mechanism gives a non-zero contribution. This arises from a wave function renormalization factor  $Z(H\bar{H})$  and the soft terms are proportional to  $|\frac{D_H W}{H}|_0 = |m + W_{mod} \frac{\bar{H}}{H}|$  which is generally non-zero. This clearly illustrates the fact that the DS mechanism is not equivalent to the usual AMSB argument.

## V. DS SUSY BREAKING AND GMSB

In the section III we rederived the DS formula for the soft masses and showed that it is different from the AMSB one if there is a mu-term. Here we will revisit the calculation and argue that it needs to be seriously modified when the field  $\chi$  is identified with the Higgs field. The reason is that the scale of the soft masses is around the scale of the Higgs vacuum expectation value. This will lead us to conclude that the problem of negative squared slepton masses that plagues AMSB is absent in the DS mechanism.

Let us first briefly review the calculation of soft masses in GMSB using the method of Giudice and Rattazzi [22]. Defining  $\alpha = g^2/4\pi$  where  $g$  is the coupling of some gauge group,

the anomalous dimension of some chiral scalar field  $Q$  is given (to one loop order) by

$$\gamma \equiv \frac{d \ln Z}{d \ln \mu} = \frac{c}{\pi} \alpha. \quad (76)$$

Here  $Z$  is the wave function renormalization of  $Q$  at the scale  $\mu$  (so that the Kaehler potential for it is  $Z(\mu)Qe^V\bar{Q}$ ) and  $c = c_2(r)$  is the quadratic Casimir for the representation  $r$ . Suppose that between the ultraviolet scale  $\Lambda$  (which could be the Planck scale or the scale associated with the hidden sector where SUSY is broken) there is an intermediate scale (messenger mass in GMSB) characterized by a chiral scalar superfield  $X$  (which can develop a non-zero vacuum expectation value (vev) and an F-term). Thus  $X = \chi$  of section III or  $X = \sqrt{\chi^+ \chi^-}$  of section II and in the MSSM should be taken to be the invariant  $X = \sqrt{H^u H^d}$ . The beta function well above and well below the scale set by the vev of  $X$  are given to one loop by  $\beta' = -b'g^3/16\pi^2$ ,  $\beta = -bg^3/16\pi^2$ . Integrating these last two equations we have

$$\alpha_X^{-1} = \alpha_\Lambda^{-1} + \frac{b'}{4\pi} \ln \frac{X\bar{X}}{\Lambda^2} \quad (77)$$

$$\alpha_\mu^{-1} = \alpha_X^{-1} + \frac{b}{4\pi} \ln \frac{\mu^2}{X\bar{X}} \quad (78)$$

Note that the coupling at the low scale  $\mu$  depends on the threshold scale  $X$ . In a supersymmetric theory the scale  $X$  should be replaced by the complete superfield and  $\alpha^{-1}$  is the real part of the chiral superfield  $f$ . Integrating (76) then gives

$$\ln Z(\mu) = \ln Z(\Lambda) + \frac{2c}{b'} \ln \frac{\alpha_\Lambda}{\alpha_X} + \frac{2c}{b} \ln \frac{\alpha_X}{\alpha_\mu} \quad (79)$$

The radiatively generated soft mass is given by

$$m_Q^2(\mu) = -\frac{\partial^2 \ln Z(\mu)}{\partial \ln X \partial \ln \bar{X}} \frac{|F_X|^2}{|X|^2} \quad (80)$$

where it is understood that the right hand side is evaluated at the vev of  $X$ . Using (79) to evaluate the derivatives and taking the limit  $\mu \rightarrow X$  (where now  $X$  is the vacuum expectation value of the field) we have

$$m_Q^2(X) = 2c \left( \frac{\alpha_X}{4\pi} \right)^2 (b - b') \frac{|F_X|^2}{|X|^2} = 2c \left( \frac{\alpha_X}{4\pi} \right)^2 (b - b') m_{3/2}^2, \quad (81)$$

where in the last step we have identified  $X$  with the field  $\chi$  of section III and ignored the 'mu' term. The beta function above the threshold has more matter states contributing than

the one below, so  $b - b'$  is always positive and (81) implies that (since  $c > 0$ ) the squared masses are always positive as in GMSB. Let us contrast this with the calculation that was done in section III, and see why it needed to be modified. There what was done was in effect to first take the limit  $\mu \rightarrow X$  in (79) in which case the last term in the right hand side of that equation disappears. Then upon doing the differentiations in (80) one gets

$$m_Q^2(X) = 2c \left( \frac{\alpha_X}{4\pi} \right)^2 b' m_{3/2}^2, \quad (82)$$

which indeed would be negative if  $b'$  is negative as is the case for the  $SU(2) \times U(1)$  group of the standard model. In other words we would have the same problem as for AMSB!

However it does not really make sense to first take the limit and then differentiate.  $\mu$  is a mass scale and may be identified with the vev of  $X$  but not with the superfield itself. On the other hand the formula (80) makes sense only when  $X$  is actually treated as the full superfield before the differentiations, and then set to its vev afterwards. Also since the scale  $X$  is to be associated with the Higgs vev and the mass scale of all other states (except the top) are below this scale the limit should be taken from below and before the  $X$  differentiation as was done above to get (81). So in approaching the SUSY breaking threshold from below it is probably appropriate to take the running as being due to the standard model states, though obviously the fact that some SUSY partners may well be below the top quark makes the precise determination of this running somewhat unclear. A detailed investigation of this will be left to a future publication.

The same modification should be made for the DS calculation of gaugino masses as well. Thus in (35)(36)(39) the coefficient  $b$  should be replaced by  $b - b'$ . It should also be noted that although (81) is independent of the Higgs vev the derivation requires the existence of a non-trivial minimum for the Higgs potential. Since in the MSSM the symmetric vacuum is only destabilized by radiative effects that depend on the breaking of supersymmetry, this mechanism depends on a bootstrap like self-consistency argument.

One also sees that (81) exhibits characteristics of both AMSB and GMSB. Like the former the squared masses are proportional to the squared gravitino mass. Furthermore like AMSB the classical contributions to the mass splittings need to vanish since otherwise they would dominate the quantum effects. This means that the supersymmetry breaking sector needs to be sequestered [3]. An example of how this could happen is the second model discussed in section IV. By contrast in GMSB the mass scale is set by the messenger mass, and the

gravitino is the lightest super-partner and one does not really need sequestering. However unlike AMSB but as in GMSB this mechanism gives positive values for all squared masses.

## VI. CONCLUSIONS

Let us summarize the main points of this paper.

- The expression for the anomaly mediated contribution to the gaugino mass is essentially contained in [8] and is given here in equation (18). When one uses the value of the F-term of the Weyl compensator that is required to get to the Einstein-Kaehler gauge, we get the formula (19) which we claim is the correct formula for the gaugino mass that can come purely from Weyl anomalies. This latter formula can alternatively be derived without going to the Weyl compensator formalism (i.e. the  $C = 1$  gauge) and in that case it comes from Jacobians associated with field redefinitions that are associated with going to the Kaehler-Einstein frame.
- An additional contribution to the gaugino mass comes from an effect noticed in [7] (DS). This when added to the previous contribution gives a formula that is superficially similar to the complete expression for the gaugino mass given in [6]. However the DS contribution can have additional terms, when there is a “mu” term in the MSSM superpotential for instance.
- There is no AMSB contribution to the soft masses. The usual argument proceeds from inserting a conformal compensator superfield factor  $C$  to multiply the ratio of scales  $\mu/\Lambda$  in the wave function renormalization  $Z(\mu/\Lambda)$ . However this ratio, being a ratio of physical scales should be independent of the Weyl gauge. Indeed inserting such a factor will violate the Weyl invariance of the formalism (which incidentally should be preserved whether or not there are mass terms in the action). Furthermore any non-trivial dependence on  $C$  in  $Z$  will mean that the former becomes a propagating field which cannot be decoupled from the action and would violate unitarity. In any case one should be able to derive a physical effect in any gauge - in particular in the usual formulation of supergravity with the Weyl compensator superfield  $C$  set to unity. This does not seem to be possible - which again suggests that the effect, at least in its original form, is absent. The point is that unless local supersymmetry is

explicitly broken by the regularization, one should be able to express the Wilsonian effective action in terms of a superpotential and an effective (quantum corrected) Kaehler potential in the standard formulation of supergravity.

- There is however a contribution which is similar to the usual AMSB one, that has been discovered by Dine and Seiberg [7]. However there are several differences. Firstly there is an additional term when there is a “mu” term present. Secondly we have shown that there are models in which the usual AMSB contribution is zero but the DS contribution is non-zero. Thirdly this DS contribution does not give rise to negative slepton squared masses. Fourthly the DS effect has nothing to do with Weyl anomalies and certainly exists independently of the particular formulation of supergravity.

## VII. ACKNOWLEDGMENTS

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## Appendix

It is helpful to consider some of the issues involved in the compensator formalism and its relation to AMSB in a simplified context. Consider the action (1) without gauge fields and with just one matter field (say  $Q$  with Kahler potential  $K = \bar{Q}Q$ ) and the Weyl compensator field  $C$ . Let us simplify further by taking these fields to be real. The bosonic part takes the form of two conformally coupled scalars and a potential term:

$$S = \frac{1}{2} \int d^4x \sqrt{g} (C^2 R + 6g^{\mu\nu} \partial_\mu C \partial_\nu C) - \int d^4x \sqrt{g} \left[ \frac{R}{6} C^2 Q^2 + g^{\mu\nu} \partial_\mu (CQ) \partial_\nu (CQ) + C^4 V(Q) \right] \quad (83)$$

This action has the Weyl invariance (descending from the super-Weyl invariance of (1))

$$g_{\mu\nu} \rightarrow e^{4\tau} g_{\mu\nu}, \quad C \rightarrow e^{-2\tau} C. \quad (84)$$

Note that the Weyl compensator  $C$  has a kinetic term with the wrong sign. However this is not a problem since it can be gauged away - it is really a spurious field which is equivalent

to a Weyl transformation. At the quantum level these transformations will have an anomaly with the structure  $\int \tau "R^2"$  where the integrand is a linear combination of four derivative terms of the metric. This will need to be cancelled by a similar term with  $\ln C$  instead of  $\tau$  that is added to the action so that the Weyl invariance is preserved at the quantum level. This is of course essentially what we did in section (II) except that there we ignored squared curvature terms and just focused on (supersymmetrized) gauge kinetic terms. This is needed for consistency since we need to be able to remove the spurious field  $C$ . The theory is completely equivalent to that with the action (83) in the gauge  $C = 1$ .

The theory is however not in Einstein frame since the scalar fields couple to curvature in the form  $\int \sqrt{g} C^2 (1 - \frac{Q^2}{3}) R$ . To go to the Einstein frame we simply pick the gauge  $C = 1/\sqrt{1 - \frac{Q^2}{3}}$  and the action (ignoring the anomaly term) becomes

$$S = \int d^4x \sqrt{g} \left[ \frac{R}{2} - \frac{1}{(1 - \frac{Q^2}{3})^2} (g^{\mu\nu} \partial_\mu Q \partial_\nu Q + V(Q)) \right] \quad (85)$$

Alternatively we could have started with the action (1) in  $C = 1$  gauge and then do a field redefinition (or equivalently a Weyl transformation)  $g_{\mu\nu} \rightarrow (1 - \frac{Q^2}{3})^{-1} g_{\mu\nu}$ . It is easily checked that this leads to the same action as (85) as it should. In the quantum theory this is not the whole story since the field redefinition results in a Jacobian factor in the path integral measure that results effectively in the same term as the one discussed earlier. The main point is that the final action including the anomaly correction must in fact be the same.  $C$  is a spurious field and can have no physical significance. This must be the case even if one integrates the fluctuations of the field down from some scale  $\Lambda$  (at which we take the above action to be a valid description) down to some lower scale  $\mu$  at which we want to investigate its physics. The corresponding renormalization constants can only depend on the ratio of scales  $\mu/\Lambda$  and clearly should not depend on the spurious field  $C$ . Any such dependence would violate the Weyl invariance which enabled us to decouple this field.

What is done in the literature on AMSB however is to break the Weyl symmetry by picking a metric - i.e. the flat metric  $\eta_{\mu\nu}$ . Once this is done of course one loses sight of the original invariance. In flat space then the Weyl invariance is replaced by conformal invariance which is broken by mass terms. Thus let us define (as is usually done in the literature)  $\hat{Q} \equiv CQ$  and take the potential to be  $V = \lambda \phi^4$ . The action (83) then becomes

$$S = \int d^4x \sqrt{g} \left[ (C^2 - \frac{\hat{Q}^2}{3}) \frac{1}{2} R + 3g^{\mu\nu} \partial_\mu C \partial_\nu C - g^{\mu\nu} \partial_\mu \hat{Q} \partial_\nu \hat{Q} - \lambda \hat{Q}^4 \right]$$



If one goes to flat space with the above metric it appears as if we have a conformally invariant flat space theory for  $\hat{Q}$  that is independent of  $C$ . Any  $C$  dependence would arise only if one had explicit mass terms. However this ignores the fact that graviton fluctuations will couple in a field dependent fashion and furthermore that  $C$  appears as a ghost. One needs to go to the Einstein frame by doing a field redefinition  $g_{\mu\nu} \rightarrow (C^2 - \hat{Q}^2/3)^{-1} g_{\mu\nu}$  and it is the new metric that should be put equal to the flat metric. This transformation however introduces  $C$  dependence into the  $\hat{Q}$  lagrangian - the potential for example becomes  $\lambda \hat{Q}^4 / (C^2 - \hat{Q}^2/3)$ .

In any case the issue is not conformal invariance. What is relevant is Weyl invariance which exists irrespective of the existence of mass terms. It is this invariance (which is manifest only if the metric is not fixed) that enables one to eliminate the spurious field  $C$ .

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  - [28] For earlier work see Dixon et al. [23] and Derendinger et al. [24]
  - [29] Bagger et al [6] actually observe that there could be additional contributions from high scale physics to their formula for the gaugino mass. Also for related work see Gaillard et al. [25].
  - [30] Note that this formula as well as similar formulae below leave the phase of the lowest component undetermined. This means only that the axionic partner of the gauge coupling has an ambiguity but it has no effect on the gaugino masses.
  - [31] It should be pointed out that the absence of the  $m_{3/2}$  term of (21) as in (19), has been shown in an explicit string theory calculation in Antoniadis and Taylor [26].
  - [32] We are essentially following an argument due to Arkani-Hamed and Murayama [27] in the global case and we give this here for completeness, even though it is not germane to our main considerations.
  - [33] In fact the argument of [4] is similar to this.
  - [34] In [7] actually the vevs  $v_{\pm}$  are taken to zero in which case, in the absence of a “mu” term the results will be completely equivalent to those of AMSB, including of course the unfortunate negative slepton mass squared result. For more details see section V.